

Knowledge and Reality B: Lecture Ten

1. Paradox One: Achilles and the Tortoise

Imagine Achilles races a tortoise over 100m. Who wins? Achilles! Imagine Achilles lets the tortoise have a 10m headstart. Who wins? Achilles? Maybe...

Says Zeno, how can Achilles win? To overtake the tortoise, Achilles must first reach the tortoise's starting point at 10m, by which time the tortoise has moved to 15m. To catch up now, Achilles has to make it to the 15m mark. But by *then* the tortoise is 17.5m along the track. To catch up now, Achilles has to make it to the 17.5m mark. But by *then* the tortoise would be 18.75m along the track – *still ahead of Achilles!* But this keeps going! Achilles will *never* catch up with the tortoise. He *always* has to get to where the tortoise was, and whenever he does *that* the tortoise is always ahead! I don't mean that you should *believe this*, but you need to explain where Zeno's reasoning has gone wrong. Harder than it looks.

2. Paradox Two: The Runner

Imagine a runner tries to cross 100m. To do that they must first cross 50m. To cross 50m they must cross 25m. To get to the 25m mark, they have to first reach 12.5m etc. Similar to the previous paradox, this never ends. So to run 100m you'd have to run *an infinite number* of distances. Or imagine that every time you ran one of those distances you counted a number. You'd have to count to infinity – which is impossible! Not just physically (as you'd find it physically impossible to count that fast) but logically.

3. Paradox Three: The Plurality Argument

There are a few of these – we'll look at just one. Take an object that can be divided into two. This can go on forever, can't it? And if it goes on forever, the object has an infinity number of parts. But if every object has an infinite number of parts, they either have a size of they don't. If they don't then you're composed of an infinite number of things with no size. But then *you* don't have any size – anything multiplied by zero has zero size! If they *do* have a size then you'd be infinitely big! *Any* size, no matter how small, multiplied by infinity would be infinitely big. So either ways, you have a problem.

4. Resolution One: Nothing Ever Changes

We might, of course, draw the lesson Zeno wants to draw. *If* things did move then we'd have a problem. Our failure to overcome the paradoxes is a *dead giveaway* that motion and plurality are all in our heads. Nothing ever moves, nothing ever changes, there's only one thing. These radical conclusions aren't *utterly* mad. Some philosophers agree with at least some of these conclusions. Spinoza believed there was only one thing, and Schaffer has defended it more recently. McTaggart had a paradox of his own to demonstrate that nothing ever changes. And philosophers throughout history have argued that the way things *seem* to be aren't how they *are* at all.

5. Resolution Two: Discrete Space and Time

All of these puzzles demand that space and time are infinitely divisible. What if that were false? What if there were a *smallest unit* of space and time? Physics might even indicate such a thing – it at least admits that there are units of space and time under which nothing makes any physical sense. If this were the case, then the paradoxes could be solved. At some point in each, there'd be a smallest unit – pretend it was 1mm.

Achilles and the Tortoise

There'd come a point at which the tortoise was 1mm ahead of Achilles. What we previously said is that Achilles crossed that 1mm gap, but the tortoise was 1.5mm ahead. Now there *is no* 1.5mm – so the tortoise just stays put. So Achilles catches up!

The Runner

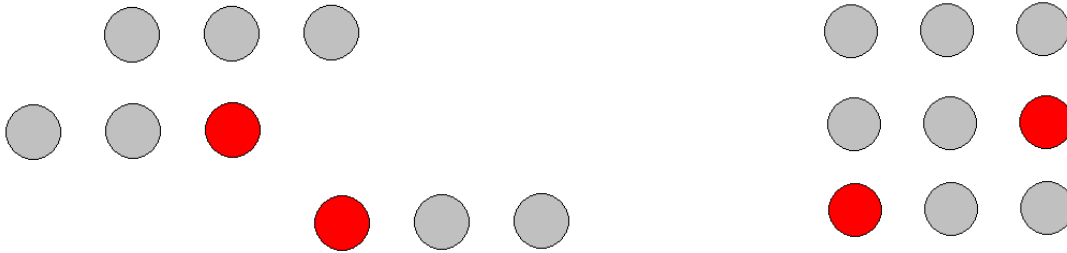
Similarly, to cross 2mm the runner will have to cross 1mm. But to cross 1mm he doesn't have to cross 0.5mm. He just crosses 1mm – it's as small as can be!

The Plurality Argument

When we get down to the 1mm sized lumps, you just *can't* get any further. So there aren't an infinite number of bits. And they're *all* 1mm in diameter – the smallest one could go.

6. Paradox Four: The Stadium

But Zeno has this nailed. Imagine a Stadium with rows of people. Now imagine they all move. Further imagine that they're moving in the smallest time possible. We get an absurdity. Concentrate on two of them. When they move, if this takes place in the smallest time possible *they never pass one another* – but surely they must do!



As such a thing is absurd, Zeno concludes that space cannot have smallest bits. If it exists at all, it must be infinitely divisible.

7. Resolution Three: Actual/Potential Infinity

Aristotle had a crack at solving the problem. There could never be an infinite number of things (he said). For it to be a number, it must be possible to count to it – but as you can't count to infinity, it can't be a number. Thus, says Aristotle, there can't be an *actually* infinite number of things. But there can be a *potentially* infinite number of things. For example, take the bar. Aristotle denies that *before* you divide it there are all these parts. If there were all these parts, there'd be an infinite number of them – *a fortiori* an actual infinite number of things. But the parts exist 'in potential' – they exist *potentially*. That is, we could (in theory) keep dividing and nothing would stop us. But even though there's never a point at which we would fail to be able to divide further *at every point there is still a finite number of things* (i.e. no actual infinity) Aristotle thinks this will solve the problems. Zeno, he thinks, is relying upon the idea that there are an *actual* infinity of parts (or distances to be run etc.)

But there are only a *potential* infinity. Similarly for the runner paradox. You manage to run 100m, even though *in theory* we could divide that 100m into other sections of distance (and such a division would never end). There's only one distance to run – the other distances only exist 'potentially'. So there *aren't* an infinite number of different distances to cover.

I leave you to think about the Aristotelian notions. But note this – maybe there *are* actual infinities. For instance, the Wilkinson Microwave Anisotropy Probe (WMAP) has given us some evidence that space is infinitely big. And if it's infinitely big the chance of there being other big bangs is quite high. The chance of there being *an infinite number* of other Big Bangs is quite high! So there'd be an infinite number of other planets, stars etc. all existing *right now* somewhere in the infinitude of space. But if Aristotle is right then this cannot be. Do we really get to tell scientists that they're wrong?